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Strength Analysis for the Adhesive Layer on the Basis of Intensity of Singular Stress

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Abstract

In this paper the strength of adhesive joint under tension and bending is discussed on the basis of intensity of singular stress by the application of FEM. A useful method is presented with focusing on the stress at the edge of interface between the adhesive and adherent obtained by FEM. After analyzing the adhesive joint strength with all material combinations, it is found that to improve the interface strength, thin adhesive layers are desirable because the intensity of singular stress decreases with decreasing of the thickness.

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Keywords: Adhesive joint, Adhesive thickness, Intensity of singular stress, Elsticity

1. Instruction

With the widely used of adhesive joint, the evaluation of adhesive joint strength also be paid more and more attention to. About the adhesive joint strength many researches have been done [1],[2], but most of them are limited in the experiments, and the most popular experiments are the adhesive joint subjected to the tension and bending, for example, the mode in Fig.1(a) could describe the micro-tensile bond test, which is a laboratory procedure frequently employed today in an attempt to predict the clinical effectiveness of adhesive used for bonding composite restorations to the dental substrate. In this paper, adhesive joint strength for Fig.1(a) and (b) will be analyzed on the basis of intensity of singular stress

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Nomenclature

G, ν	shear modulus and Poisson's ratio
r, θ	polar coordinates around the interface edge
W, l, t	dimensionless of the adhesive joint, W the width, l the height, t the adhesive thickness
K_σ, F_σ	intensity of singular stress and dimensionless intensity of singular stress
α, β	Dunders' parameters which are expressed by Possion's ratio ν and shear modulus G

using the solutions for the reference problem shown in Fig.2 (a) and (b).

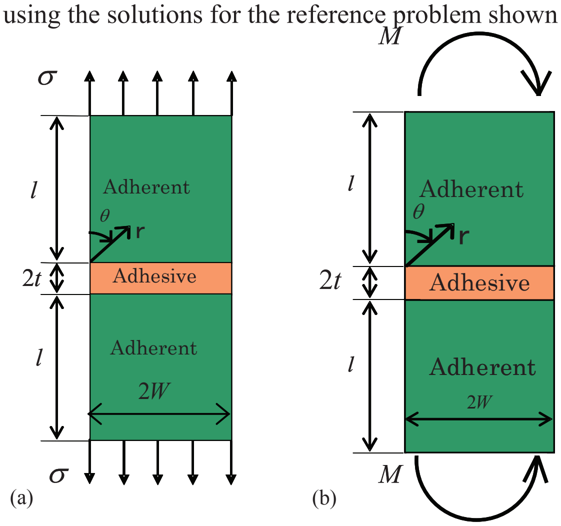

Fig.1 Adhesive joint under (a) tension, (b) bending

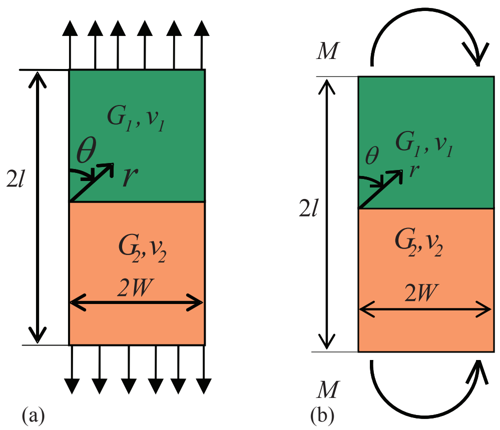

Fig.2 Bonded strip under (a) tension, (b) bending

2. Analysis Method

For the adhesive joint as shown in Fig.1, it is known that the interface stress $\sigma_{ij}\,(ij = rr, \theta\theta, r\theta)$ goes to infinity at the edge of joint and has singularity of $\sigma_{ij} \propto 1/r^{1-\lambda}$ when $\alpha(\alpha - 2\beta) > 0$; Besides, when $\theta = \pi/2$, the singularity of the stress λ at the joint of interface can be expressed by the following equation [3], [4].

$$\left[\sin^2\left(\frac{\pi}{2}\lambda\right) - \lambda^2\right]^2 \beta^2 + 2\lambda^2\left[\sin^2\left(\frac{\pi}{2}\lambda\right) - \lambda^2\right]\alpha\beta + \lambda^2\left(\lambda^2 - 1\right)\alpha^2 + \frac{\sin^2(\lambda\pi)}{4} = 0 \qquad (1)$$

$$\alpha = \frac{G_1(\kappa_2 + 1) - G_2(\kappa_1 + 1)}{G_1(\kappa_2 + 1) + G_2(\kappa_1 + 1)} \qquad \beta = \frac{G_1(\kappa_2 - 1) - G_2(\kappa_1 - 1)}{G_1(\kappa_2 + 1) + G_2(\kappa_1 + 1)}$$

$$\kappa_j = \begin{cases} \dfrac{3 - \nu_j}{1 + \nu_j}(plane \quad stress) \\ 3 - 4\nu_j(plane \quad strain) \end{cases}, \kappa_j = \quad (j = 1, 2) \qquad (2)$$

The intensity of singular stress K_σ at the adhesive dissimilar joint is expressed as

$$K_\sigma = \lim_{r\to 0}\left[r^{1-\lambda} \times \sigma_{\theta|\theta=\pi/2}(r) \right] \tag{3}$$

and the dimensionless of intensity of singular stress F_σ is defined by the following equation.

$$F_\sigma = \frac{K_\sigma}{\sigma(2W)^{1-\lambda}} = \frac{\lim_{r\to 0}[r^{1-\lambda}\sigma_{\theta|\theta=\pi/2}(r)]}{\sigma(2W)^{1-\lambda}} \quad \text{(for tension)}$$

$$F_\sigma = \frac{K_\sigma}{(6M/4W^2)(2W)^{1-\lambda}} = \frac{\lim_{r\to 0}[r^{1-\lambda}\sigma_{\theta|\theta=\pi/2}(r)]}{(6M/4W^2)(2W)^{1-\lambda}} \quad \text{(for bending)} \tag{4}$$

Here, σ is the stress applyed to the y direction.

In this paper, the finite element method is used to obtain the stress at the joint of interface, and the software is MSC.MARC 2007. The width of the model is W=1000mm, and the length l is 2W, because it is demonstrated that when $l \geq 2W$, the interface stresses are the same. The adhesive thickness t/W is changed as 0.001, 0.01, 0.1, 0.5, 1, 2, 4.

We will propose the method of calculating the intensity of singular stress from the results of FEM. In this paper, the ratio of intensity of singular stress K^1_σ / K^2_σ will be considered. Here, the superscripts 1,2 mean that t/W is different. As shown in Eqs. (3),(4), the dimensionless intensity of singular stress is related to the distance r, singular index λ, and stress σ or M, width W and limiting stress $\lim_{r\to o}\sigma_{\theta|\theta=\pi/2}$. Considering different adhesive thicknesses t_1, t_2 as problem 1 and problem 2, both of which have the same stress at infinity σ or M and material combinations. Therefore, it should be noted that the singular index $\lambda_1 = \lambda_2$. As shown in Eq. (5), the ratio of intensity of singular stress K^1_σ / K^2_σ is only related to the ratio of stress $\lim_{r\to o}(\sigma^1_{\theta|\theta=\pi/2} / \sigma^2_{\theta|\theta=\pi/2})$ depending of adhesive thickness t, and the ratio of intensity of singular stress is expressed as shown in Eq. (5).

$$\frac{K^1_\sigma}{K^2_\sigma} = \frac{\sigma^1(2W)^{1-\lambda_1}F^1_\sigma}{\sigma^2(2W)^{1-\lambda_1}F^2_\sigma} = \frac{F^1_\sigma}{F^2_\sigma} = \lim_{r\to 0}\frac{\left[r^{1-\lambda_1}\ \sigma^1_{\theta|\theta=\pi/2}(r)\right]}{\left[r^{1-\lambda_1}\ \sigma^2_{\theta|\theta=\pi/2}(r)\right]} = \lim_{r\to 0}\frac{\sigma^1_{\theta|\theta=\pi/2}(r)}{\sigma^2_{\theta|\theta=\pi/2}(r)} \quad \text{(for tension)}$$

$$\frac{K^1_\sigma}{K^2_\sigma} = \frac{(6M/4W^2)^1(2W)^{1-\lambda_1}F^1_\sigma}{(6M/4W^2)^2(2W)^{1-\lambda_1}F^2_\sigma} = \frac{F^1_\sigma}{F^2_\sigma} = \lim_{r\to 0}\frac{\left[r^{1-\lambda_1}\ \sigma^1_{\theta|\theta=\pi/2}(r)\right]}{\left[r^{1-\lambda_1}\ \sigma^2_{\theta|\theta=\pi/2}(r)\right]} = \lim_{r\to 0}\frac{\sigma^1_{\theta|\theta=\pi/2}(r)}{\sigma^2_{\theta|\theta=\pi/2}(r)} \quad \text{(for bending)} \tag{5}$$

Therefore, in this paper, only stress $\lim_{r\to o}\sigma_{\theta|\theta=\pi/2}$ is calculated. In this paper, the ratio of intensity of singular stress is mainly considered in the analysis. To obtain the intensity of singular stress from the ratio, the reference problem as shown in Fig.2 will be used.

3. Intensity of singular stress for bonded strip as a reference solution

However, to obtain the intensity of singular stress, a reference solution is necessary. Chen-Nisitani [5] and Noda et. al [6] have analyzed the intensity of singular stress in a bonded strip under tension and bending in Fig.2 accurately by using the body force method. In the previous studies, only the results for singular stress $\lambda \leq 1$ are indicated, and the results are showed in Fig.3. However, in this study, all material combinations are newly considered and, therefore Fig.3 includes new results for $F_\sigma > 1$ where no singular stress because singular index $\lambda > 1$.

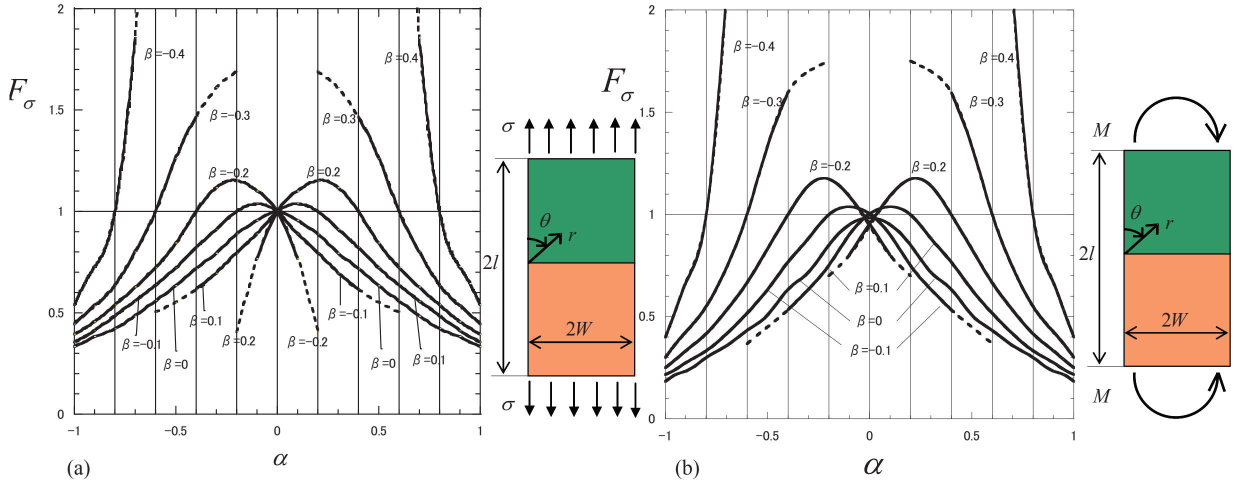

Fig.3 F_σ for a bonded strip in Fig.2 (a) tension , (b) bending

4. Results for the Adhesive joint

From the analysis method, it is found that the ratio of intensity of singular stress can be given very accurately independent of mesh size, and only the values for the first element are needed as the ratio of stresses keeps the constant along the interface when $r \to 0$ [7]. However, because of the space of the paper, it will not be illustrated here.

4.1. Results for the adhesive joint under tension

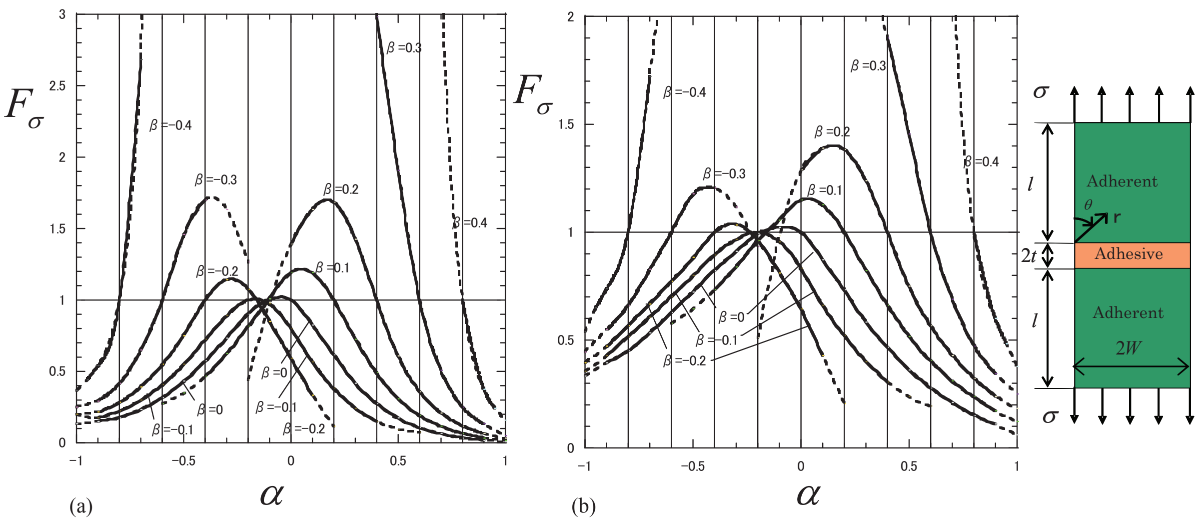

Fig.4 F_σ with varying material combination β in Fig.1 (a) when (a) $t/W = 0.001$; (b) $t/W = 0.1$

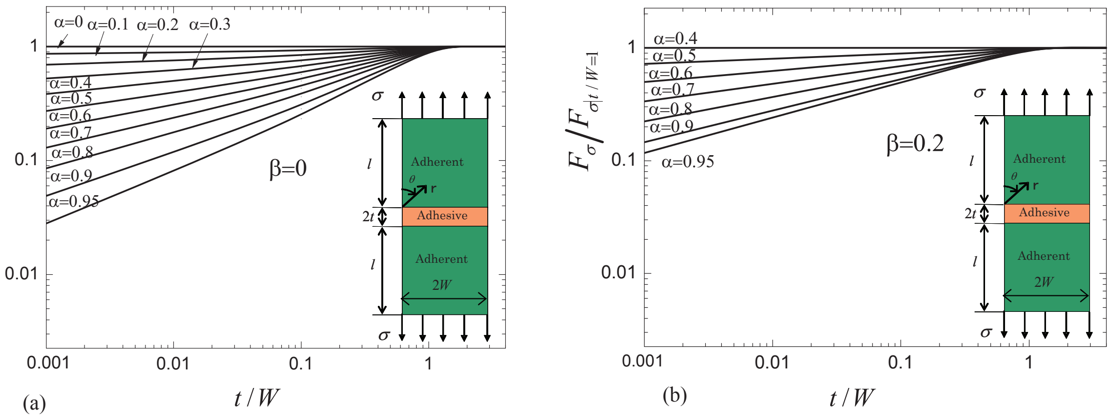

Fig.5 $F_\sigma / F_{\sigma|t/W=1}$ with varying adhesive thickness t/W in Fig.1 (a) for (a) $\beta = 0$, (b) $\beta = 0.2$

Figure 4 shows F_σ with varying α and β when (a) $t/W = 0.001$; (b) $t/W = 0.1$. It can be seen that F_σ increases with increasing of α when α is small. On the other hand, F_σ decreases with increasing of α when α is large. Comparing the results of $t/W = 0.001$ and $t/W = 0.1$, it is found that the increase and decrease of F_σ for $t/W = 0.001$ is quick, while the increase and decrease of F_σ for $t/W = 0.1$ is slower. Moreover, the variation range for $t/W = 0.001$ is large, while the variation range for $t/W = 0.1$ is smaller.

Generally, the Young's modulus E_2 of adhesive is smaller than the Young's modulus E_1 of adherent: $E_2 \le E_1$, and the Poisson's ratio ν_2 of adhesive is larger than the Poisson's ratio ν_1 of adherent: $\nu_2 \ge \nu_1$. In this case, it is found that $\alpha \ge 0$ and $\alpha - 2\beta \ge 0$ and therefore singularity stress exists around the edge of the interface. Figure 5 shows examples of the variation of logarithmic $F_\sigma / F_{\sigma|t/W=1}$ and t/W for $\beta = 0$ and for $\beta = 0.2$ satisfying $\alpha \ge 0$ and $\alpha - 2\beta > 0$. It is found that $F_\sigma / F_{\sigma|t/W=1}$ increases with increasing t/W until

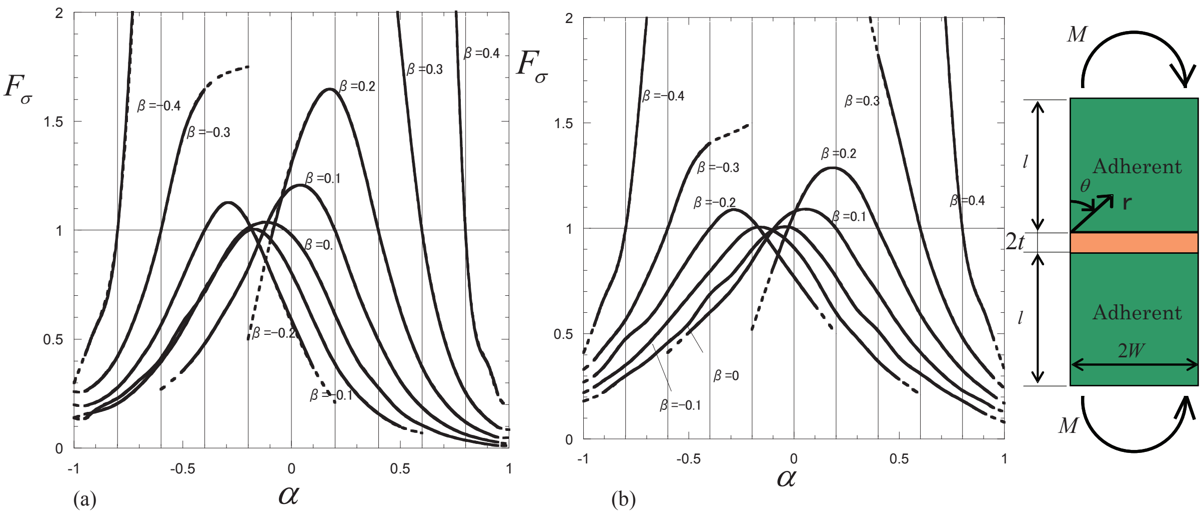

Fig.6 F_σ with varying material combination β in Fig.(b) when (a) $t/W = 0.001$; (b) $t/W = 0.1$

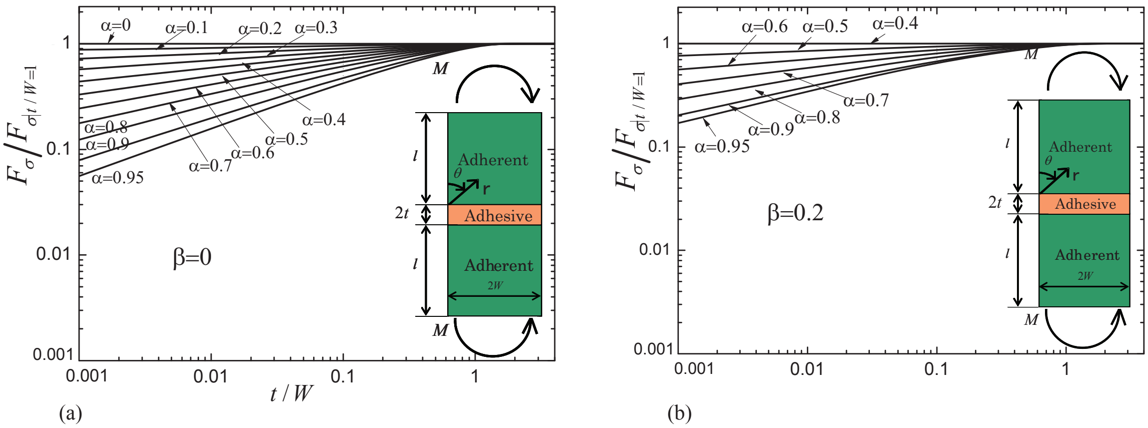

Fig.7 $F_{\sigma}/F_{\sigma|t/W=1}$ with varying adhesive thickness t/W in Fig.(b) for (a) $\beta = 0$, (b) $\beta = 0.2$

$t/W = 1$ for all the material combinations. To improve the interface strength, thin adhesive layers are desirable because the intensity of singular stress decreases with decreasing the thickness.

4.2 Results for the adhesive joint under bending

When the adhesive joint is under bending, results of F_{σ} and the effect of adhesive thickness and material combinations on it have been obtained using the same analysis method. Figure 6 shows variations F_{σ} with all material combinations when (a) $t/W = 0.001$; (b) $t/W = 0.1$ and Fig.7 shows examples of the variation of logarithmic $F_{\sigma}/F_{\sigma|t/W=1}$ and t/W for $\beta = 0$ and $\beta = 0.2$ satisfying $\alpha \geq 0$ and $\alpha - 2\beta > 0$. Comparing the results of tension and bending, it should be noticed that F_{σ} for bending take larger comparing with F_{σ} for tension when $t/W = 0.001$ and $t/W = 0.1$, although F_{σ} for tension always take larger comparing with F_{σ} for bending for the boned strip.

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